

## 7-8 Inverse Functions and Relations

Desmos Activity builder

Warm-up!

$$f(x) = 2x$$

$$g(x) = x^3 + x^2 + 2$$

$$[g \circ f](x) = 8x^3 + 4x^2 + 2$$

$$g(2x) = (2x)^3 + (2x)^2 + 2$$

45.

$$[h \circ (g \circ f)](2)$$

$$f(x) = 4x$$

$$g(x) = 2x - 1$$

$$h(x) = x^2 + 1$$

$$f(2) = 4(2) = 8$$

$$g(8) = 15$$

$$h(15) = 226$$

Property of Inverse Functions

If  $f(a) = b$ , then  $f^{-1}(b) = a$ .

$$f(x) = 3x - 1 \quad f(3) = 8$$

$$f^{-1}(x) = \frac{1}{3}x + \frac{1}{3} \quad f^{-1}(8) = 3$$

Finding the inverse.

ex:

$$f(x) = \frac{-1}{2}x + 1$$

$$y = -\frac{1}{2}x + 1$$

$$x = -\frac{1}{2}y + 1$$

$$x - 1 = -\frac{1}{2}y$$

$$-2x + 2 = y$$

$$f^{-1}(x) = -2x + 2$$

1. change to  $y =$
2. switch  $x$  and  $y$
3. solve for  $y$
4. change to  $f^{-1}(x) =$

Find the inverse:

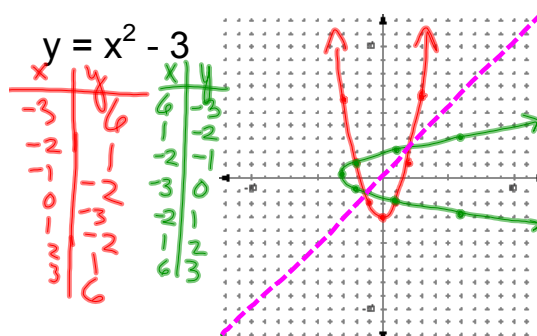
$$1. f(x) = \frac{3}{4}x - 6 \quad f^{-1}(x) = \frac{4}{3}x + 8$$

$$2. g(x) = \frac{x+2}{3} \quad g^{-1}(x) = 3x - 2$$

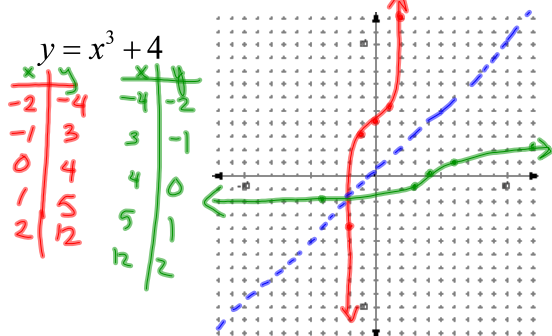
$$3. f(x) = x^3 - 3 \quad f^{-1}(x) = \sqrt[3]{x+3}$$

After finding the inverse, graph #3 and the inverse on your calc or desmos, but open a new desmos window. (not student.desmos)

Sketch the inverse.



Sketch the inverse.



Show that two functions are inverses of one another

Two functions are inverses of each other iff the composition is the identity function.

$$I(x) = x$$

$$[f \circ g](x) = x$$

$$[g \circ f](x) = x$$

Show  $f(x) = \frac{-1}{2}x + 1$  and  $g(x) = -2x + 2$  are inverses.

$$\begin{aligned}
 [f \circ g](x) &= f(-2x + 2) \\
 &= -\frac{1}{2}(-2x + 2) + 1 \\
 &= x - 1 + 1 \\
 [f \circ g](x) &= x \quad \checkmark \\
 [g \circ f](x) &= g\left(\frac{-1}{2}x + 1\right) \\
 &= -2\left(\frac{-1}{2}x + 1\right) + 2 \\
 [g \circ f](x) &= x - 2 + 2 \\
 [g \circ f](x) &= x \quad \checkmark
 \end{aligned}$$

Determine whether each pair of functions are inverse functions. (use composition to do this.)

1.  $h(x) = 5x - 7$       2.  $f(x) = 6x + 2$

$g(x) = \frac{1}{5}(x + 7)$        $g(x) = x - \frac{1}{3}$

$$\begin{aligned}
 [h \circ g](x) &= h\left(\frac{1}{5}(x + 7)\right) \\
 &= 5\left(\frac{1}{5}(x + 7)\right) - 7 \\
 &= x + 7 - 7 \\
 [h \circ g](x) &= x \quad \checkmark \\
 [g \circ h](x) &= g(5x - 7) \\
 &= \frac{1}{5}(5x - 7 + 7) \\
 &= \frac{1}{5}(5x) \\
 [g \circ h](x) &= x \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 [f \circ g](x) &= f\left(x - \frac{1}{3}\right) \\
 &= 6\left(x - \frac{1}{3}\right) + 2 \\
 &= 6x - 2 + 2 \\
 [f \circ g](x) &= 6x
 \end{aligned}$$

Not Inverses

Back to desmos activity to graph composition of functions.

HW p393  
15-27 odd  
32, 33